

Fast Obligation Translation and Synthesis

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Marcin Jurdzinski

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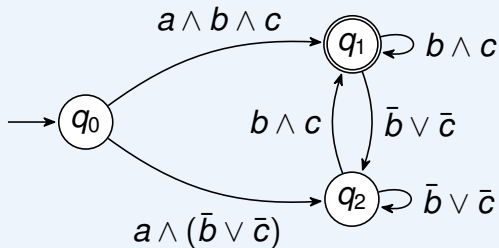
Moshe Y. Vardi

Shufang Zhu

MTBDDs Against the Alphabet-Explosion Problem

Consider a DBA for $a \wedge GF(b \wedge c)$:

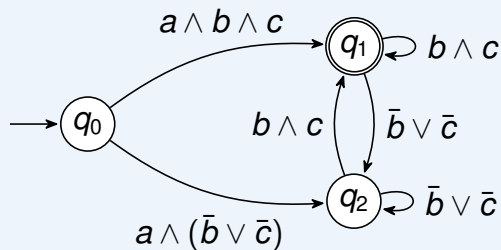
An Explicit Automaton



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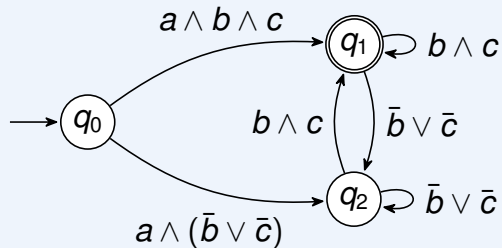


Labeling with Boolean formulas
to avoid using the alphabet $2^{\{a,b,c\}}$

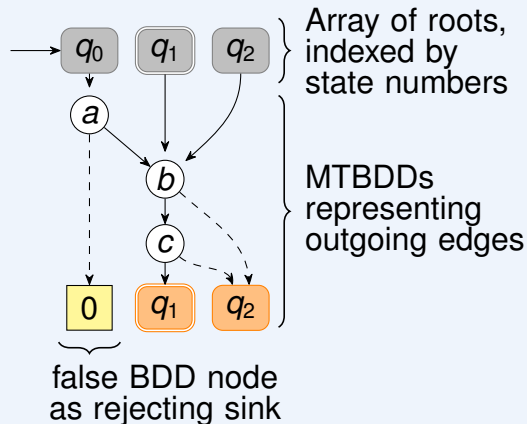
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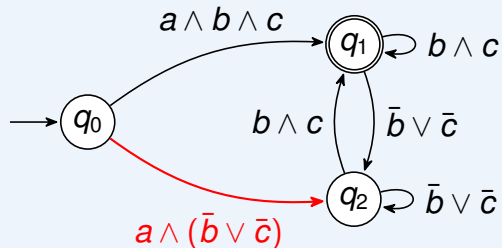
A Semi-Symbolic Encoding



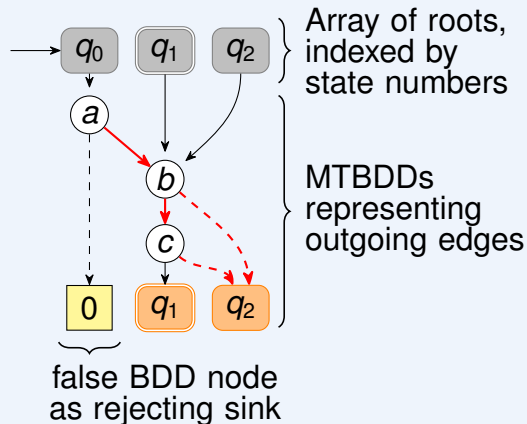
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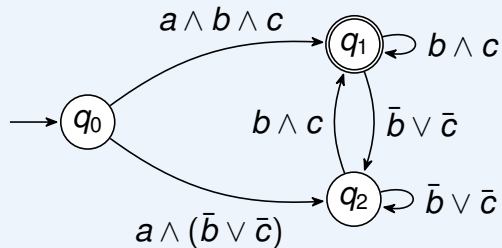
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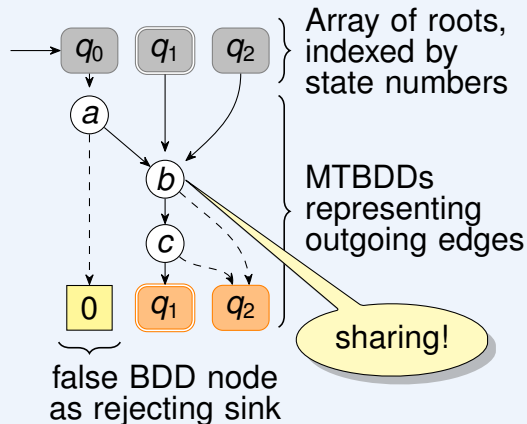
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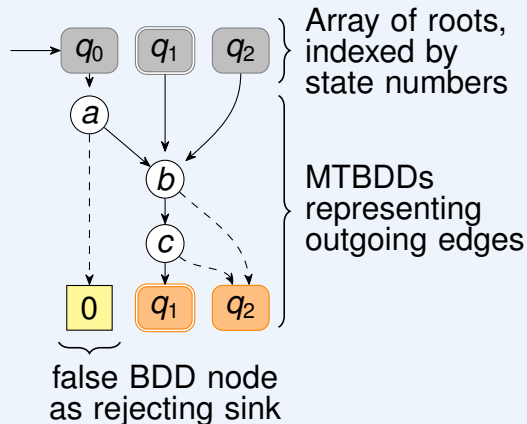
MTBDDs Against the Alphabet-Explosion Problem

Originally used by Mona to implement:

- ▶ DFAs over propositional alphabets,
- ▶ Boolean operations over DFAs,
- ▶ Minimization of DFAs.

Successfully used by
several LTL_f synthesis tools.

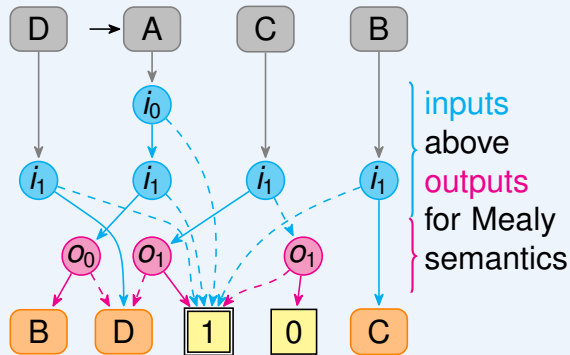
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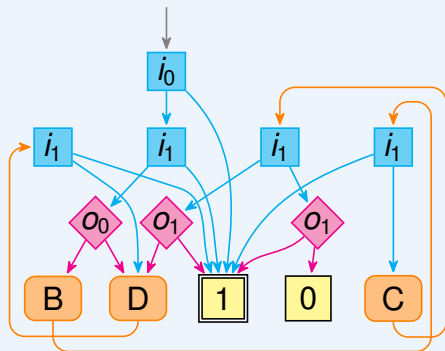
Previous Work: Spot's `ltlfsynt` Tool

- ▶ On-the-fly translation of LTL_f to MTBDD-based DFA,
- ▶ Interpretation of the MTBDD-based DFA directly as a reachability game,
- ▶ Game solved on-the-fly during its construction.

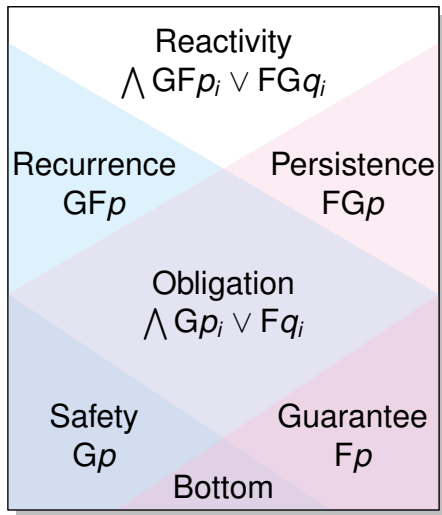
The DFA



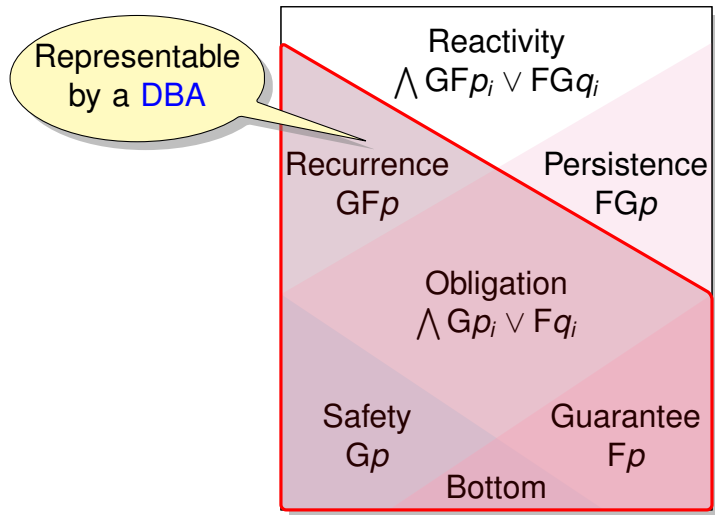
The Game Interpretation



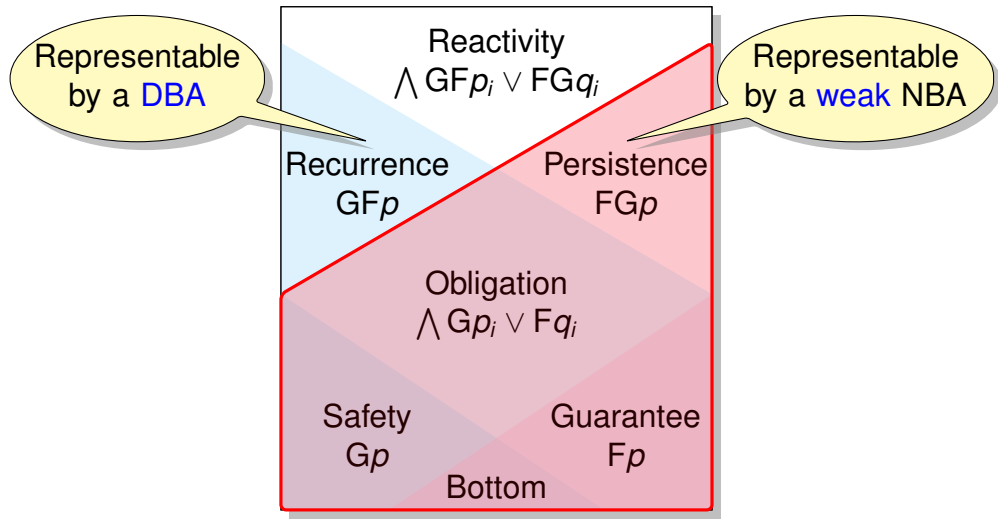
Going Back to LTL and ω -Automata



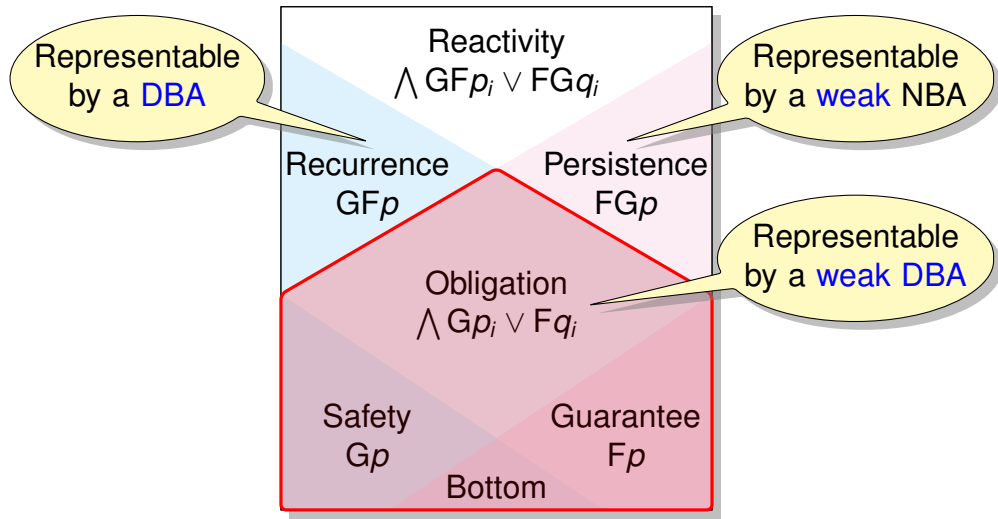
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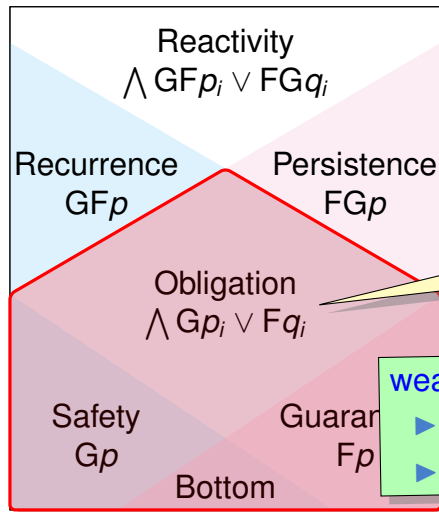
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Going Back to LTL and ω -Automata

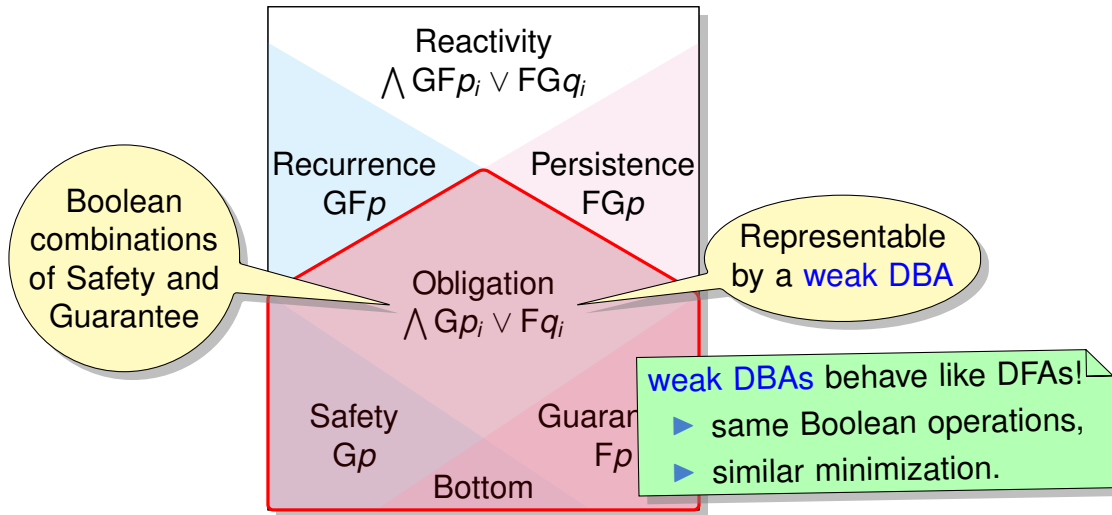


Representable
by a **weak DBA**

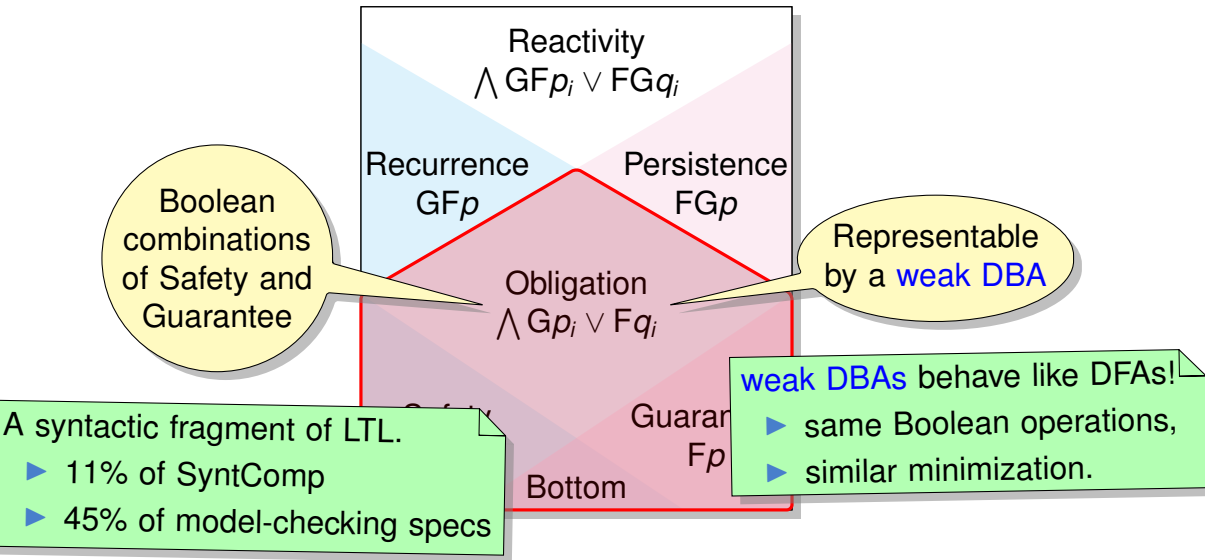
weak DBAs behave like DFAs!

- ▶ same Boolean operations,
- ▶ similar minimization.

Going Back to LTL and ω -Automata



Going Back to LTL and ω -Automata



Contributions

A new translation from syntactic obligations to weak DBA

- ▶ Produces an **MTBDD-encoded weak DBA** directly
- ▶ States are labeled by obligation formulas
- ▶ Acceptance of a state is decided by looking at the Boolean combination of safety and guarantee properties its formula represents.
- ▶ **Minimization is cheap** in practice.

Faster reactive synthesis for syntactic obligations

- ▶ Translate the specification to MTBDD **on-the-fly**...
- ▶ ... while solving a **weak game** (linear)

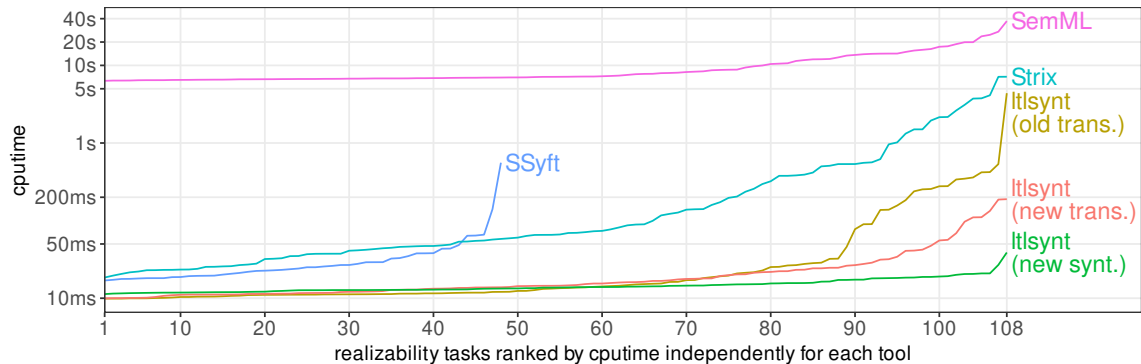
All of this is implemented in Spot 2.15



▶ [www](http://www.spot-logic.com)

Improving `ltlsynt` on Obligations

The SyntComp repository contains 108 obligations.



Conclusion

- ▶ “Obligation” as an **interesting fragment** of LTL:

- ▶ Translation from obligations to WDBA is easy.
- ▶ WDBA can be minimized in polynomial time.
- ▶ Weak games can be solved in linear time.

contrib.

one optimization in paper

- ▶ **MTBDD** to represent **deterministic** automata:

- ▶ A compact way to cope with propositional alphabets.
- ▶ With the right order, can be interpreted as a game directly.

contrib.

- ▶ **Implemented** in Spot 2.15



▶ [www](http://www.spot-logic.com)

- ▶ Translation from obligations to MTBDD-based WDBA.
- ▶ Minimization of WDBA using MTBDDs.
- ▶ On-the-fly synthesis (not just realizability) using MTBDDs.

contrib.

contrib.

contrib.

Fast Obligation Synthesis

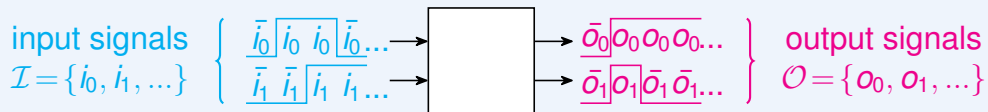
- 1. Title
- 2. MTBDD
- 3. CIAA'25
- 4. Hierarchy
- 5. Contributions
- 6. Speedup
- 7. Conclusion

Extra Slides

- 9. Synthesis Problem
- 10. Synthesis Pipeline
- 11. MTDBA
- 12. $LTL \rightarrow MTBDD$
- 13. Obligations
- 14. λ
- 15. Game view
- 16. Propositional Equivalence
- 17. Obligations Frequency
- 18. $LTL \rightarrow MTWDBA$
- 19. Weak Games
- 20. On-the-fly

Reactive Synthesis in a Nutshell

A **reactive controller** produces output as a reaction to its input



LTL reactive synthesis problems

Given an LTL specification relating **input signals** and **output signals** over time:

Realizability: decide if a controller exists;

Synthesis: construct it (e.g., as an And-Inverter Graph).

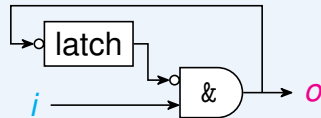
our focus

Textbook Reactive Synthesis

1. LTL Spec.

$$i \leftrightarrow F(o)$$

6. Output ALG



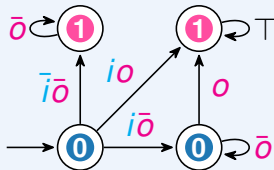
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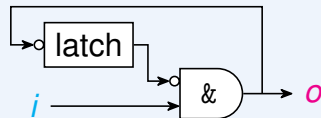
$$i \leftrightarrow F(o)$$

2. DPA (max odd)

$$\text{Fin}(\textcircled{0}) \vee \text{Inf}(\textcircled{1})$$



6. Output ALG



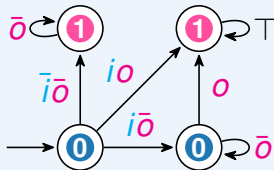
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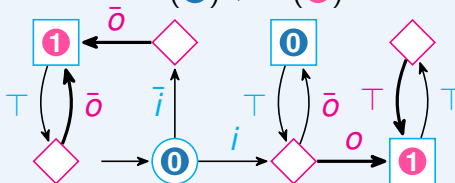
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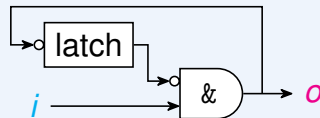


3. Parity Game (max odd)

$$\text{Fin}(\textcircled{0}) \vee \text{Inf}(\textcircled{1})$$



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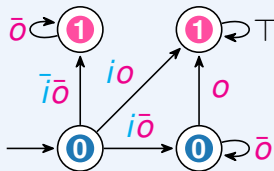
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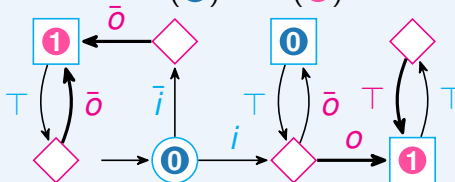
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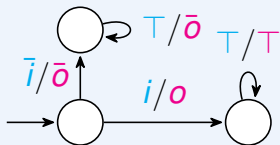


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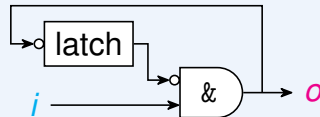
$$\text{Fin}(\textcircled{0}) \vee \text{Inf}(\textcircled{1})$$



4. Winning Strategy



6. Output ALG



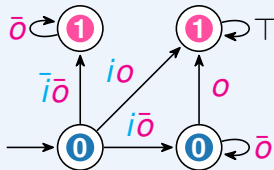
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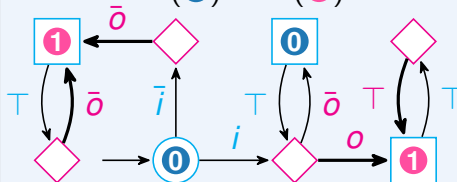
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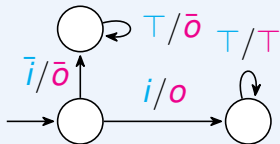


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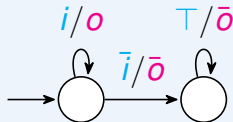
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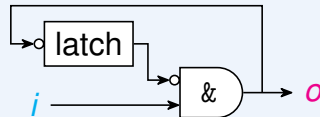
4. Winning Strategy



5. Simplified Strategy



6. Output ALG



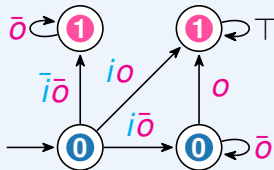
Textbook Reactive Realizability

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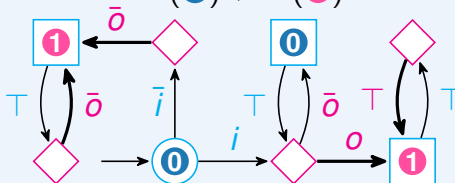
2. DPA (max odd)

$$\text{Fin}(\mathbf{0}) \vee \text{Inf}(\mathbf{1})$$



3. Parity Game (max odd)

$$\text{Fin}(\mathbf{0}) \vee \text{Inf}(\mathbf{1})$$



4. Result

REALIZABLE

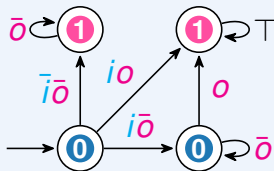
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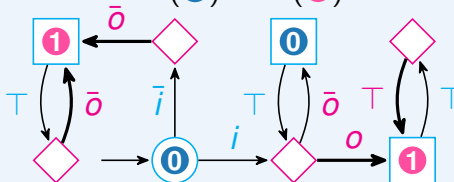
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4. Result

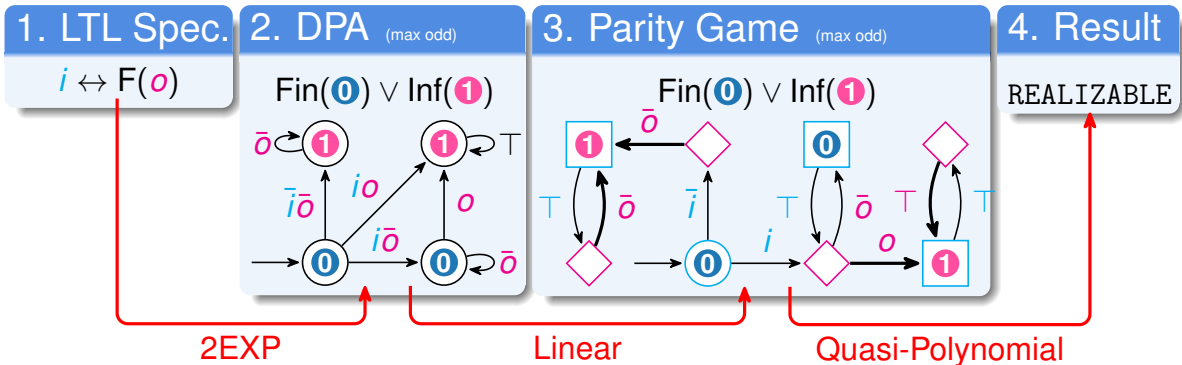
REALIZABLE

2EXP

Linear

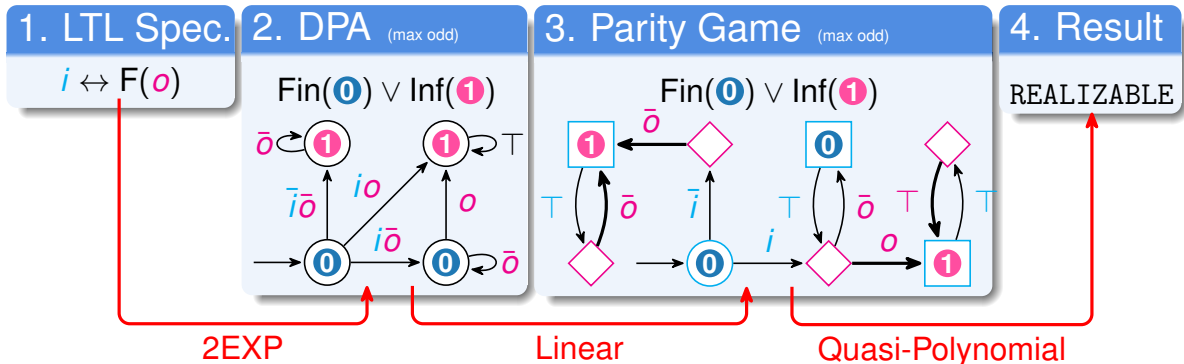
Quasi-Polynomial

Textbook Reactive Realizability



Contribution: faster pipeline for the “syntactic obligation” fragment of LTL.

Reactive Realizability



Contribution: faster pipeline for the “syntactic obligation” fragment of LTL.

MTBDD-based repr. of Weak DBA,
usable directly as a game

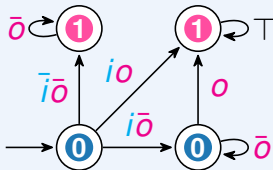
Reactive Realizability

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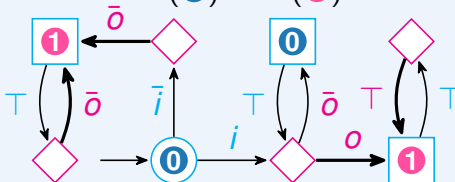
$$\text{Fin}(\mathbf{0}) \vee \text{Inf}(\mathbf{1})$$



2EXP

3. Parity Game (max odd)

$$\text{Fin}(\mathbf{0}) \vee \text{Inf}(\mathbf{1})$$



Linear

Quasi-Polynomial

4. Result

REALIZABLE

Contribution: faster pipeline for the “syntactic obligation” fragment of LTL.

MTBDD-based repr. of Weak DBA,
usable directly as a game

2EXP

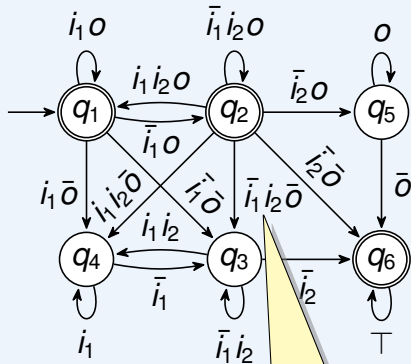
(but simpler, faster)

Linear

(weak game solving)

MTBDD-based Representation of a DBA

An “explicit” DBA



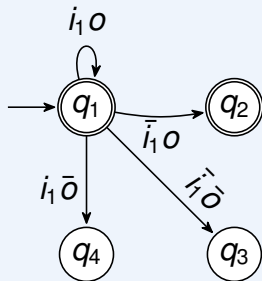
A “semi-symbolic” DBA

alphabet explosion problem:

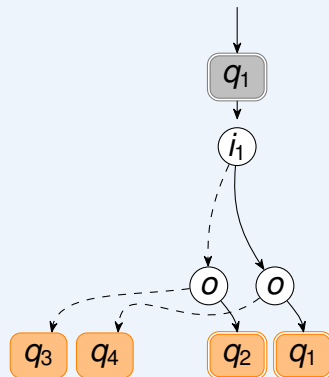
$$\Sigma = 2^{\mathcal{I} \cup \mathcal{O}}$$

MTBDD-based Representation of a DBA

An “explicit” DBA

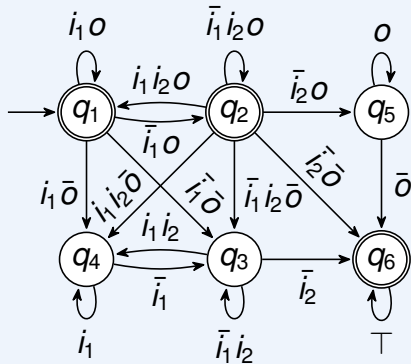


A “semi-symbolic” DBA

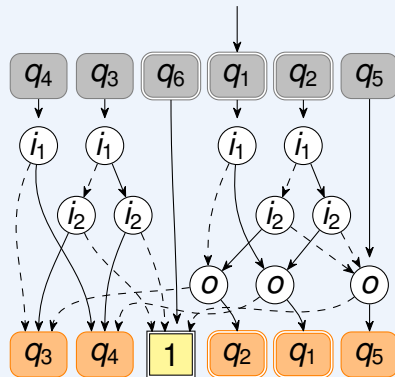


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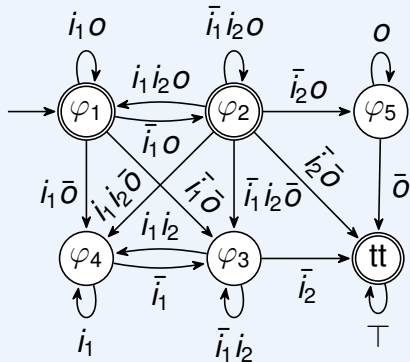


A “semi-symbolic” DBA



MTBDD-based Representation of an LTL-labeled DBA

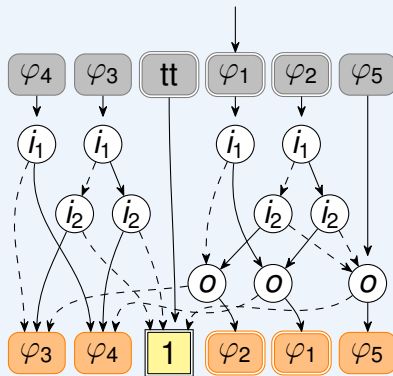
An “explicit” DBA



$$\varphi_1 = G(i_1 \vee Xi_2) \leftrightarrow Go$$

$$\varphi_2 = (i_2 \wedge G(i_1 \vee Xi_2)) \leftrightarrow Go$$

A “semi-symbolic” DBA

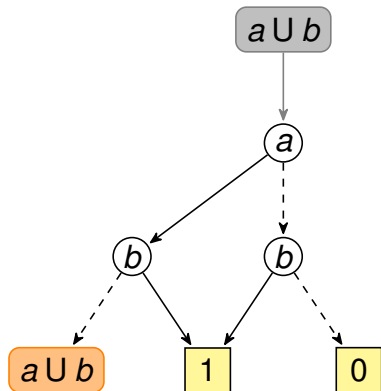


$$\varphi_3 = \neg(i_2 \wedge G(i_1 \vee Xi_2))$$

$$\varphi_4 = \neg G(i_1 \vee Xi_2)$$

$$\varphi_5 = \neg Go$$

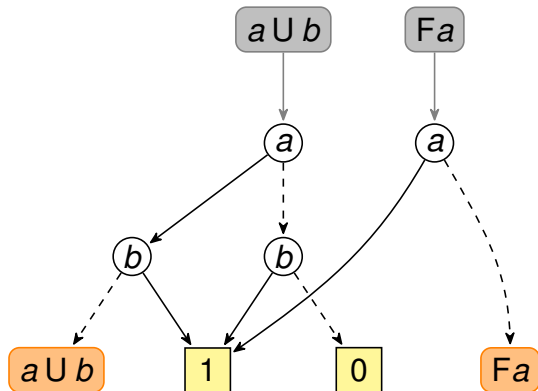
From LTL to MTBDD



This is a deterministic representation of the *next normal form* (XNF):

$$a \cup b \equiv b \vee (a \wedge X(a \cup b))$$

From LTL to MTBDD

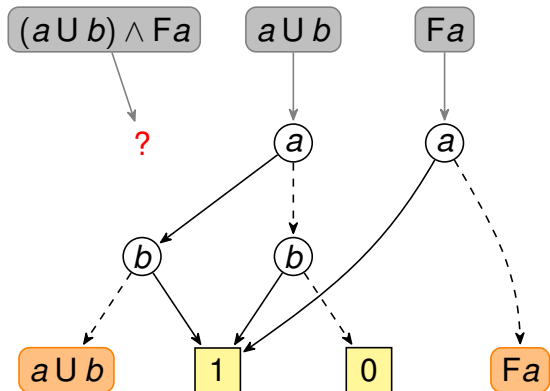


This is a deterministic representation of the *next normal form* (XNF):

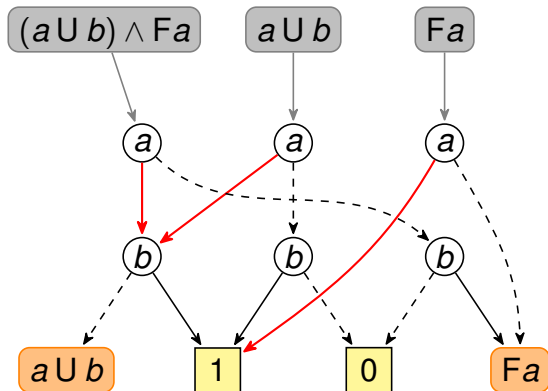
$$a \mathbf{U} b \equiv b \vee (a \wedge X(a \mathbf{U} b))$$

$$F a \equiv a \vee X F a$$

From LTL to MTBDD

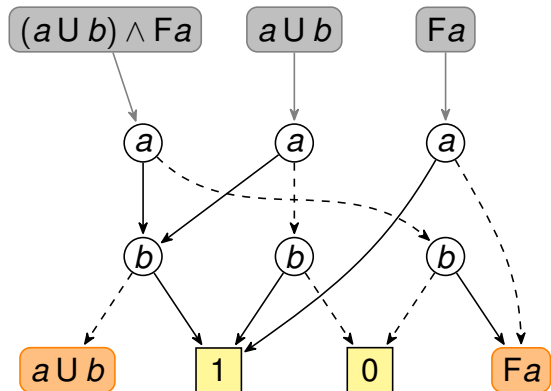


From LTL to MTBDD



Classical BDD apply procedure, but combine terminals with “ $\wedge$ ”.
(Leaves **0** and **1** can help shortcut the recursion.)

From LTL to MTBDD



We are just missing a way to decide which states are accepting.

This is where the restriction to **obligations** matters.

Obligations = Weak Deterministic Büchi Automata

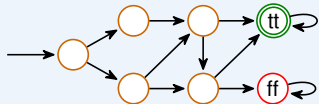
Bottom (Safety $\cap$ Guarantee)

$$\varphi_B ::= \text{ff} \mid \text{tt} \mid p \mid \neg\varphi_B \mid \varphi_B \wedge \varphi_B \mid \varphi_B \vee \varphi_B \\ \mid X\varphi_B$$

Obligations = Weak Deterministic Büchi Automata

Bottom (Safety $\cap$ Guarantee)

$\varphi_B ::= \text{ff} \mid \text{tt} \mid p \mid \neg\varphi_B \mid \varphi_B \wedge \varphi_B \mid \varphi_B \vee \varphi_B$
 $\mid X\varphi_B$

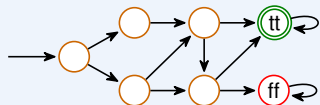


DBA is a DAG,
until $\textcircled{\text{tt}}$ or $\textcircled{\text{ff}}$

Obligations = Weak Deterministic Büchi Automata

Bottom (Safety $\cap$ Guarantee)

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DBA is a DAG,
until tt or ff

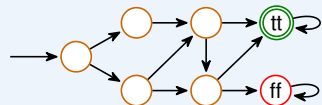
Safety

$$\varphi_S ::= \varphi_B \mid \varphi_S \wedge \varphi_S \mid \varphi_S \vee \varphi_S \\ \mid X\varphi_S \mid \mathbf{G}\varphi_S \mid \varphi_S \mathbf{R} \varphi_S$$

Obligations = Weak Deterministic Büchi Automata

Bottom (Safety $\cap$ Guarantee)

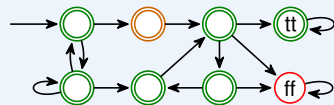
$\varphi_B ::= \text{ff} \mid \text{tt} \mid p \mid \neg\varphi_B \mid \varphi_B \wedge \varphi_B \mid \varphi_B \vee \varphi_B$
 $\mid X\varphi_B$



DBA is a DAG,
until tt or ff

Safety

$\varphi_S ::= \varphi_B \mid \varphi_S \wedge \varphi_S \mid \varphi_S \vee \varphi_S$
 $\mid X\varphi_S \mid G\varphi_S \mid \varphi_S R \varphi_S$

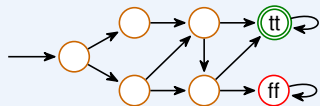


all states but ff
are accepting

Obligations = Weak Deterministic Büchi Automata

Bottom (Safety $\cap$ Guarantee)

$$\varphi_B ::= \text{ff} \mid \text{tt} \mid p \mid \neg \varphi_B \mid \varphi_B \wedge \varphi_B \mid \varphi_B \vee \varphi_B \\ \mid X\varphi_B$$



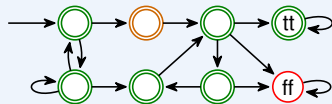
DBA is a DAG,
until tt or ff

Guarantee

$$\varphi_G ::= \varphi_B \mid \varphi_G \wedge \varphi_G \mid \varphi_G \vee \varphi_G \\ \mid X\varphi_G \mid F\varphi_G \mid \varphi_G \text{ U } \varphi_G$$

Safety

$$\varphi_S ::= \varphi_B \mid \varphi_S \wedge \varphi_S \mid \varphi_S \vee \varphi_S \\ \mid X\varphi_S \mid G\varphi_S \mid \varphi_S \text{ R } \varphi_S$$

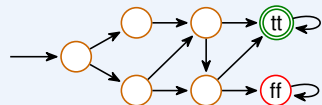


all states but ff
are accepting

Obligations = Weak Deterministic Büchi Automata

Bottom (Safety $\cap$ Guarantee)

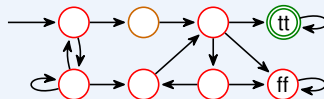
$$\varphi_B ::= \text{ff} \mid \text{tt} \mid p \mid \neg\varphi_B \mid \varphi_B \wedge \varphi_B \mid \varphi_B \vee \varphi_B \\ \mid X\varphi_B$$



DBA is a DAG,
until tt or ff

Guarantee

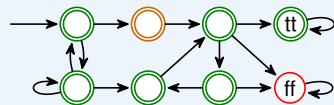
$$\varphi_G ::= \varphi_B \mid \varphi_G \wedge \varphi_G \mid \varphi_G \vee \varphi_G \\ \mid X\varphi_G \mid F\varphi_G \mid \varphi_G \text{ U } \varphi_G$$



all states but tt
are rejecting

Safety

$$\varphi_S ::= \varphi_B \mid \varphi_S \wedge \varphi_S \mid \varphi_S \vee \varphi_S \\ \mid X\varphi_S \mid G\varphi_S \mid \varphi_S \text{ R } \varphi_S$$

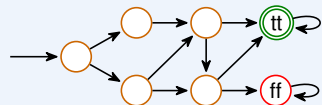


all states but ff
are accepting

Obligations = Weak Deterministic Büchi Automata

Bottom (Safety $\cap$ Guarantee)

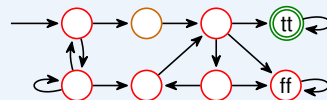
$$\varphi_B ::= \text{ff} \mid \text{tt} \mid p \mid \neg\varphi_B \mid \varphi_B \wedge \varphi_B \mid \varphi_B \vee \varphi_B \\ \mid X\varphi_B$$



DBA is a DAG,
until $\textcircled{\text{tt}}$ or $\textcircled{\text{ff}}$

Guarantee

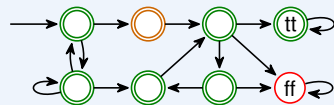
$$\varphi_G ::= \varphi_B \mid \varphi_G \wedge \varphi_G \mid \varphi_G \vee \varphi_G \mid \neg\varphi_S \\ \mid X\varphi_G \mid F\varphi_G \mid \varphi_G \text{ U } \varphi_G$$



all states but $\textcircled{\text{tt}}$
are rejecting

Safety

$$\varphi_S ::= \varphi_B \mid \varphi_S \wedge \varphi_S \mid \varphi_S \vee \varphi_S \mid \neg\varphi_G \\ \mid X\varphi_S \mid G\varphi_S \mid \varphi_S \text{ R } \varphi_S$$

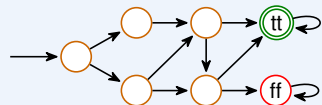


all states but $\textcircled{\text{ff}}$
are accepting

Obligations = Weak Deterministic Büchi Automata

Bottom (Safety $\cap$ Guarantee)

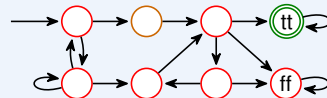
$$\varphi_B ::= \text{ff} \mid \text{tt} \mid p \mid \neg\varphi_B \mid \varphi_B \wedge \varphi_B \mid \varphi_B \vee \varphi_B \\ \mid X\varphi_B$$



DBA is a DAG,
until tt or ff

Guarantee

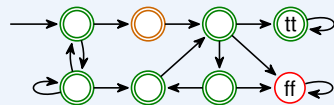
$$\varphi_G ::= \varphi_B \mid \varphi_G \wedge \varphi_G \mid \varphi_G \vee \varphi_G \mid \neg\varphi_S \\ \mid X\varphi_G \mid F\varphi_G \mid \varphi_G \text{ U } \varphi_G$$



all states but tt
are rejecting

Safety

$$\varphi_S ::= \varphi_B \mid \varphi_S \wedge \varphi_S \mid \varphi_S \vee \varphi_S \mid \neg\varphi_G \\ \mid X\varphi_S \mid G\varphi_S \mid \varphi_S \text{ R } \varphi_S$$



all states but ff
are accepting

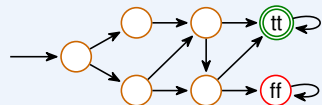
Obligation

$$\varphi_O ::= \varphi_G \mid \varphi_S \mid \neg\varphi_O \mid \varphi_O \wedge \varphi_O \mid \varphi_O \vee \varphi_O \\ \mid X\varphi_O$$

Obligations = Weak Deterministic Büchi Automata

Bottom (Safety $\cap$ Guarantee)

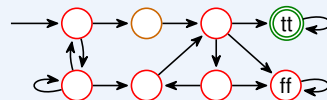
$$\varphi_B ::= \text{ff} \mid \text{tt} \mid p \mid \neg\varphi_B \mid \varphi_B \wedge \varphi_B \mid \varphi_B \vee \varphi_B \\ \mid X\varphi_B$$



DBA is a DAG,
until $\textcircled{\text{tt}}$ or $\textcircled{\text{ff}}$

Guarantee

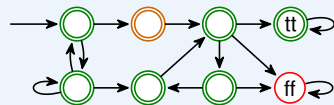
$$\varphi_G ::= \varphi_B \mid \varphi_G \wedge \varphi_G \mid \varphi_G \vee \varphi_G \mid \neg\varphi_S \\ \mid X\varphi_G \mid F\varphi_G \mid \varphi_G \text{ U } \varphi_G$$



all states but $\textcircled{\text{tt}}$
are rejecting

Safety

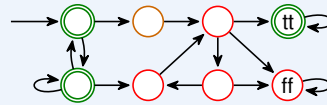
$$\varphi_S ::= \varphi_B \mid \varphi_S \wedge \varphi_S \mid \varphi_S \vee \varphi_S \mid \neg\varphi_G \\ \mid X\varphi_S \mid G\varphi_S \mid \varphi_S \text{ R } \varphi_S$$



all states but $\textcircled{\text{ff}}$
are accepting

Obligation

$$\varphi_O ::= \varphi_G \mid \varphi_S \mid \neg\varphi_O \mid \varphi_O \wedge \varphi_O \mid \varphi_O \vee \varphi_O \\ \mid X\varphi_O$$

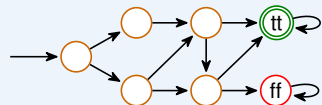


each SCC
contains only
accepting or
rejecting states

Obligations = Weak Deterministic Büchi Automata

Bottom (Safety $\cap$ Guarantee)

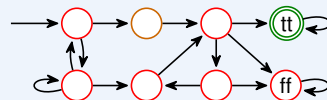
$$\varphi_B ::= \text{ff} \mid \text{tt} \mid p \mid \neg\varphi_B \mid \varphi_B \wedge \varphi_B \mid \varphi_B \vee \varphi_B \\ \mid X\varphi_B$$



DBA is a DAG,
until tt or ff

Guarantee

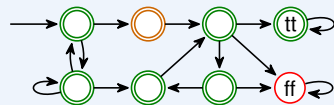
$$\varphi_G ::= \varphi_B \mid \varphi_G \wedge \varphi_G \mid \varphi_G \vee \varphi_G \mid \neg\varphi_S \\ \mid X\varphi_G \mid F\varphi_G \mid \varphi_G \text{ U } \varphi_G$$



all states but tt
are rejecting

Safety

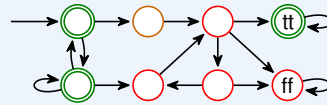
$$\varphi_S ::= \varphi_B \mid \varphi_S \wedge \varphi_S \mid \varphi_S \vee \varphi_S \mid \neg\varphi_G \\ \mid X\varphi_S \mid G\varphi_S \mid \varphi_S \text{ R } \varphi_S$$



all states but ff
are accepting

Obligation

$$\varphi_O ::= \varphi_G \mid \varphi_S \mid \neg\varphi_O \mid \varphi_O \wedge \varphi_O \mid \varphi_O \vee \varphi_O \\ \mid X\varphi_O \mid \varphi_O \text{ U } \varphi_G \mid \varphi_O \text{ R } \varphi_S$$



each SCC
contains only
accepting or
rejecting states

Deciding acceptance of an obligation-labeled state

$$\lambda(\text{ff}) = \perp$$

$$\lambda(\text{tt}) = \top$$

$$\lambda(\mathbf{G}\alpha) = \top$$

$$\lambda(\alpha \mathbf{R} \beta) = \top$$

$$\lambda(\mathbf{F}\alpha) = \perp$$

$$\lambda(\alpha \mathbf{U} \beta) = \perp$$

$$\lambda(\neg\alpha) = \neg\lambda(\alpha)$$

$$\lambda(\alpha \wedge \beta) = \lambda(\alpha) \wedge \lambda(\beta)$$

$$\lambda(\alpha \vee \beta) = \lambda(\alpha) \vee \lambda(\beta)$$

Deciding acceptance of an obligation-labeled state

$$\lambda(\text{ff}) = \perp$$

$$\lambda(\text{tt}) = \top$$

$$\lambda(p) = *$$

$$\lambda(X\alpha) = *$$

$$\lambda(\neg\alpha) = \neg\lambda(\alpha)$$

$$\lambda(G\alpha) = \top$$

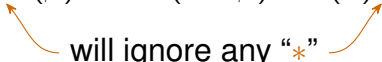
$$\lambda(\alpha \text{ R } \beta) = \top$$

$$\lambda(F\alpha) = \perp$$

$$\lambda(\alpha \text{ U } \beta) = \perp$$

$$\lambda(\alpha \wedge \beta) = \lambda(\alpha) \wedge \lambda(\beta) \quad \lambda(\alpha \vee \beta) = \lambda(\alpha) \vee \lambda(\beta)$$

will ignore any “*”



Deciding acceptance of an obligation-labeled state

$$\lambda(\text{ff}) = \perp$$

$$\lambda(\mathbf{G}\alpha) = \top$$

$$\lambda(\mathbf{F}\alpha) = \perp$$

$$\lambda(\text{tt}) = \top$$

$$\lambda(\alpha \mathbf{R} \beta) = \top$$

$$\lambda(\alpha \mathbf{U} \beta) = \perp$$

$$\lambda(p) = *$$

$$\lambda(\mathbf{X}\alpha) = *$$

$$\lambda(\neg\alpha) = \neg\lambda(\alpha)$$

$$\lambda(\alpha \wedge \beta) = \lambda(\alpha) \wedge \lambda(\beta)$$

$$\lambda(\alpha \vee \beta) = \lambda(\alpha) \vee \lambda(\beta)$$

will ignore any “*”

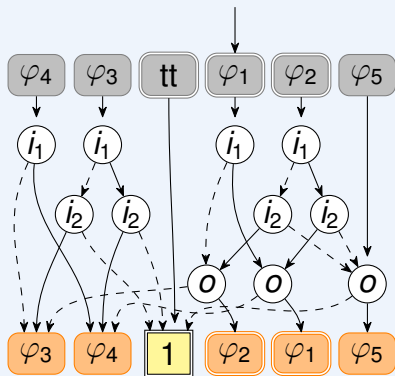
Example:

$$\lambda(\varphi_2) = \lambda((\underbrace{i_2}_{*} \wedge \underbrace{\mathbf{G}(i_1 \vee \mathbf{X}i_2)}_{\top})) \leftrightarrow \underbrace{\mathbf{Go}}_{\top} = \top$$

follows from the above rules

MTBDD-based Representation of an LTL-labeled DBA

A “semi-symbolic” WDBA



$$\varphi_1 = G(i_1 \vee Xi_2) \leftrightarrow Go$$

$$\varphi_2 = (i_2 \wedge G(i_1 \vee Xi_2)) \leftrightarrow Go$$

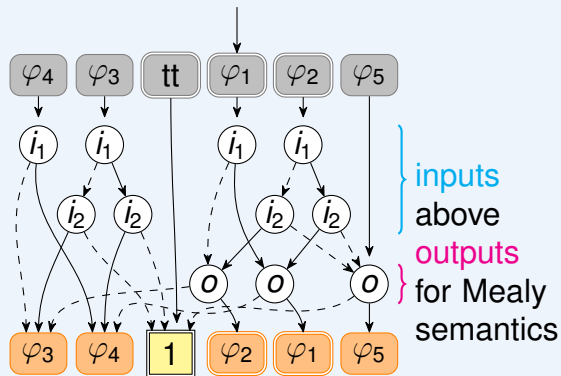
$$\varphi_3 = \neg(i_2 \wedge G(i_1 \vee Xi_2))$$

$$\varphi_4 = \neg G(i_1 \vee Xi_2)$$

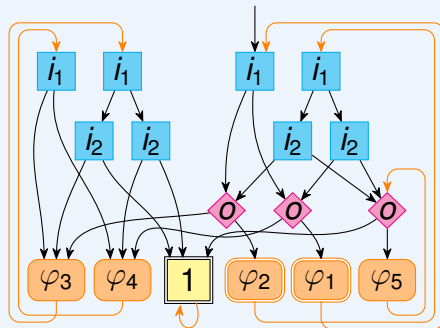
$$\varphi_5 = \neg Go$$

Game Interpretation Is Straightforward

A “semi-symbolic” WDBA

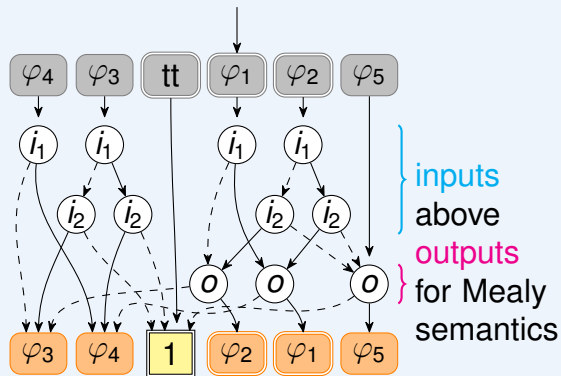


Weak Game Interpretation

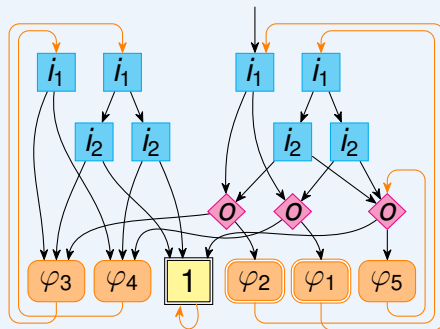


Game Interpretation Is Straightforward

A “semi-symbolic” WDBA

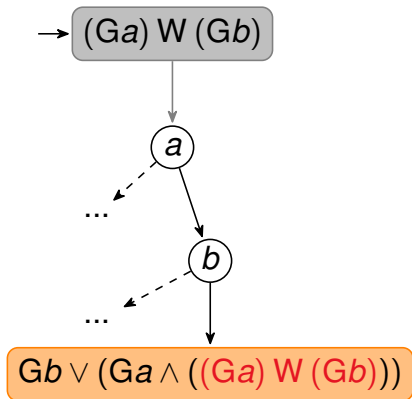


Weak Game Interpretation

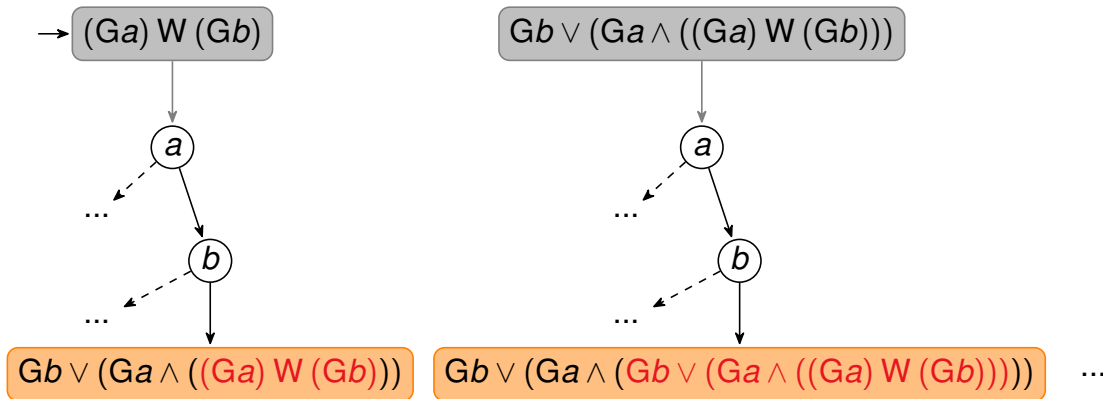


Weak games can be solved in linear time by enumerating SCCs bottom-up.
The whole translation+game solving is implemented on-the-fly.

Propositional Equivalence

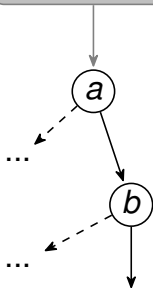


Propositional Equivalence



Propositional Equivalence

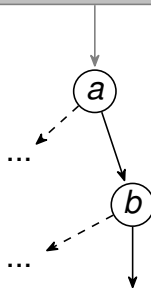
→ $(Ga) \vee (Gb)$



$Gb \vee (Ga \wedge ((Ga) \vee (Gb)))$

$p_3 \vee (p_2 \wedge p_1)$

$Gb \vee (Ga \wedge ((Ga) \vee (Gb)))$

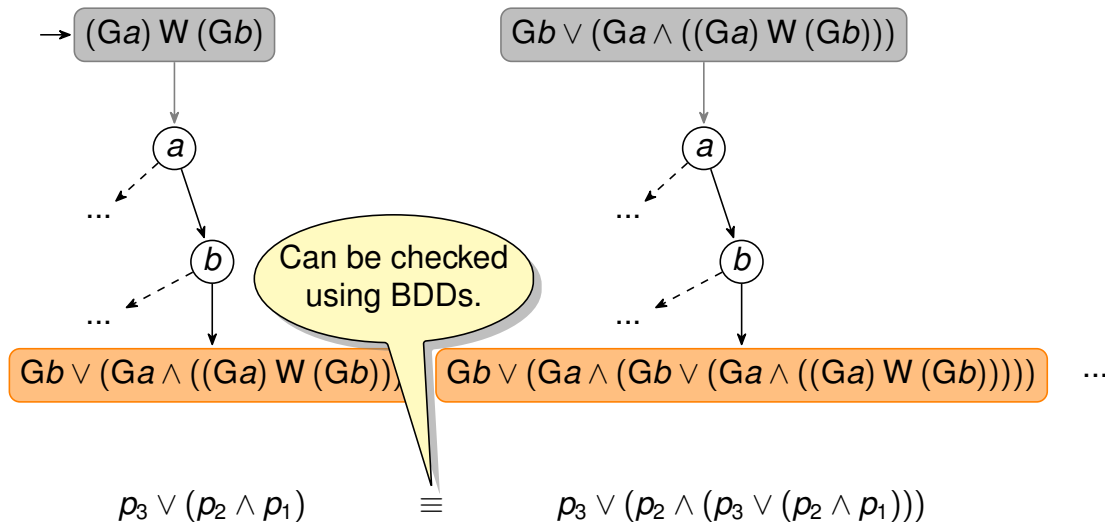


$Gb \vee (Ga \wedge (Gb \vee (Ga \wedge ((Ga) \vee (Gb)))))$

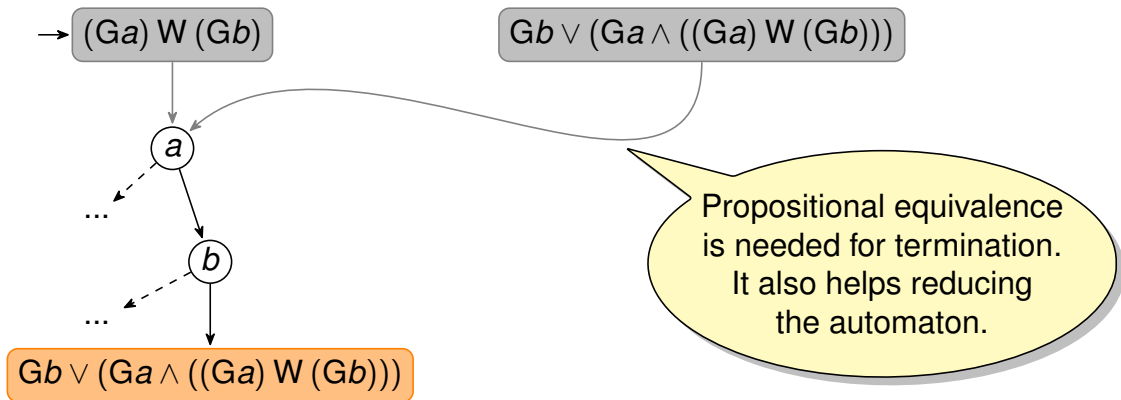
...

$p_3 \vee (p_2 \wedge (p_3 \vee (p_2 \wedge p_1)))$

Propositional Equivalence



Propositional Equivalence



Obligations Frequency

source	set size	property classes					syntactic classes				
		O	$O \setminus S \setminus G$	$S \setminus B$	$G \setminus B$	B	O	$O \setminus S \setminus G$	$S \setminus B$	$G \setminus B$	B
SyntComp	959	≥ 262	≥ 172	≥ 73	≥ 7	≥ 7	110	59	48	1	2
Dwyer et al.	55	40	2	37	1		25	13	11	1	
Somenzi&Bloem	27	16	2	6	6	2	13	4	4	5	
Etessami&Holzmann	12	5			4	1	5			5	
BEEM	20	10	1	6	1	2	7	2	4	1	
Liberouter	55	30		27	1	2	26	1	23	2	

-  Jacobs et al. *The reactive synthesis competition (SYNTCOMP): 2018–2021*. [arXiv, 2022](#). [doi](#)
-  Dwyer, Avrunin, and Corbett. *Property specification patterns for finite-state verification*. **FMSP'98**. [doi](#)
-  Somenzi and Bloem. *Efficient Büchi automata for LTL formulae*. **CAV'00**. [doi](#)
-  Etessami and Holzmann. *Optimizing Büchi automata*. **Concur'00**. [doi](#)
-  Pelánek. *BEEM: benchmarks for explicit model checkers*. **Spin'07**. [doi](#)
-  Holeček et al. *Verification results in Liberouter project*. **Technical Report 03, CESNET, 2004**

MTBDD translation is straightforward

$$\text{tr}(\text{ff}) = \boxed{0}$$

$$\text{tr}(\text{tt}) = \boxed{1}$$

$$\text{tr}(p) = \begin{array}{c} \textcircled{p} \\ \swarrow \quad \searrow \\ \boxed{0} \quad \boxed{1} \end{array} \quad \text{for } p \in \mathcal{I} \cup \mathcal{O}$$

$$\text{tr}(X\alpha) = \boxed{\alpha}$$

$$\text{tr}(\neg\alpha) = \neg\text{tr}(\alpha)$$

LTL operator

MTBDD operator

$$\text{tr}(\alpha \odot \beta) = \text{tr}(\alpha) \odot \text{tr}(\beta) \text{ for any } \odot \in \{\wedge, \vee, \rightarrow, \leftrightarrow, \oplus\}$$

$$\text{tr}(\alpha \text{ U } \beta) = \text{tr}(\beta) \vee (\text{tr}(\alpha) \wedge \boxed{\alpha \text{ U } \beta})$$

$$\text{tr}(\text{F}\alpha) = \text{tr}(\alpha) \vee \boxed{\text{F}\alpha}$$

$$\text{tr}(\alpha \text{ R } \beta) = \text{tr}(\beta) \wedge (\text{tr}(\alpha) \vee \boxed{\alpha \text{ R } \beta})$$

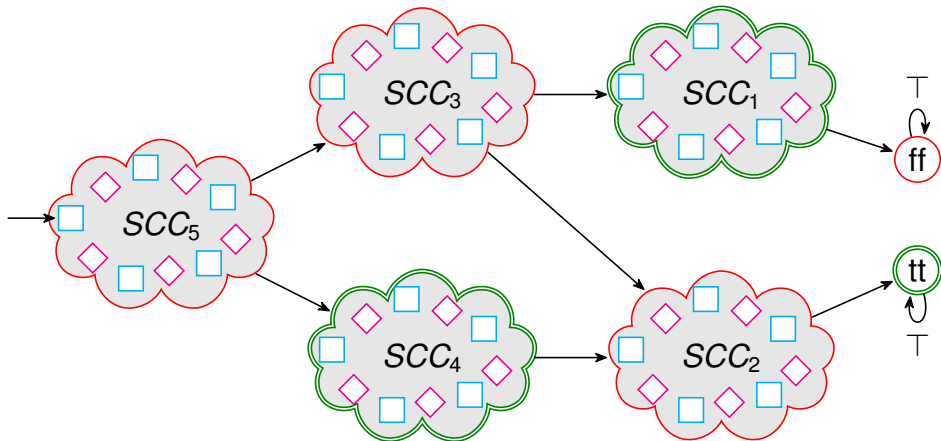
$$\text{tr}(\text{G}\alpha) = \text{tr}(\alpha) \wedge \boxed{\text{G}\alpha}$$

Acceptance of a state labeled by α is $\lambda(\alpha)$.

Solving Weak Games Bottom-Up

The **input player** wants the play to get stuck in a **rejecting SCC**.

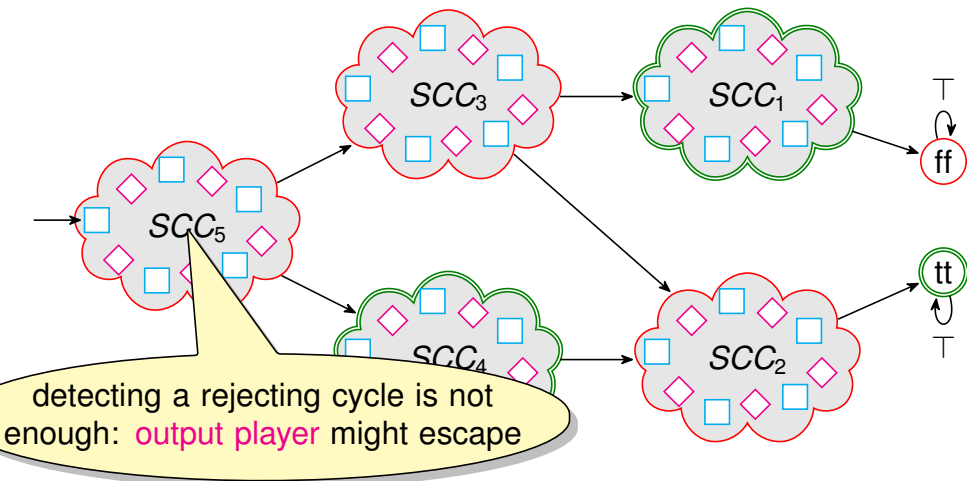
The **output player** wants the play to get stuck in an **accepting SCC**.



Solving Weak Games Bottom-Up

The **input player** wants the play to get stuck in a **rejecting SCC**.

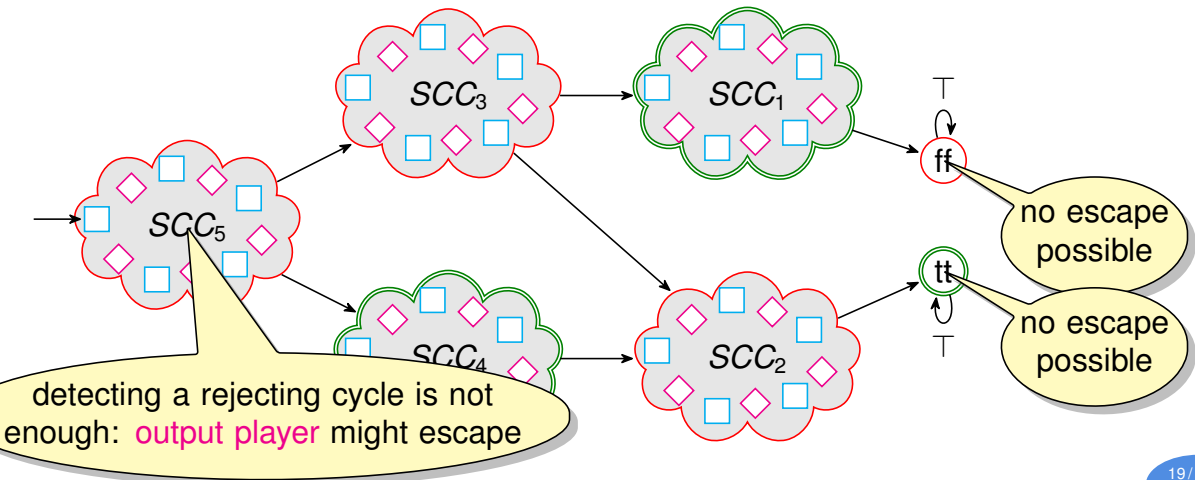
The **output player** wants the play to get stuck in an **accepting SCC**.



Solving Weak Games Bottom-Up

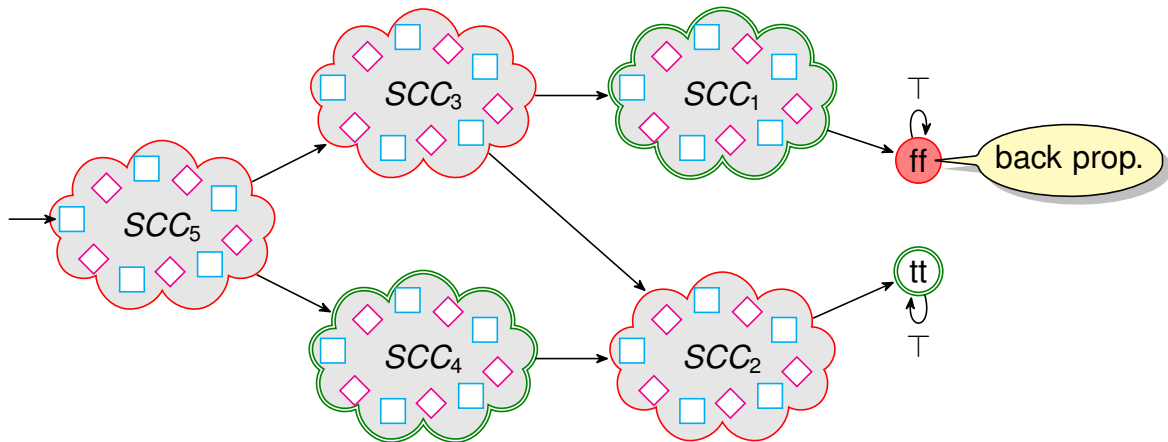
The **input player** wants the play to get stuck in a **rejecting SCC**.

The **output player** wants the play to get stuck in an **accepting SCC**.



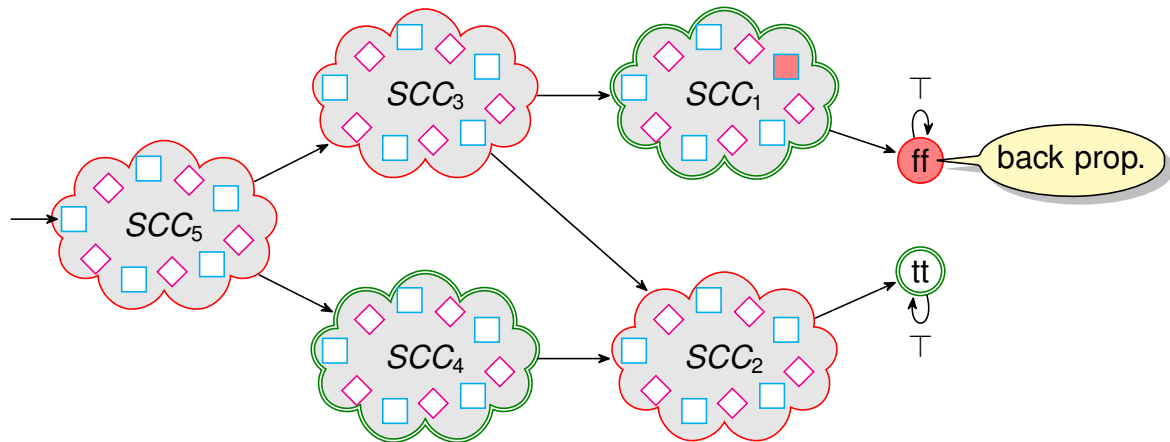
Solving Weak Games Bottom-Up

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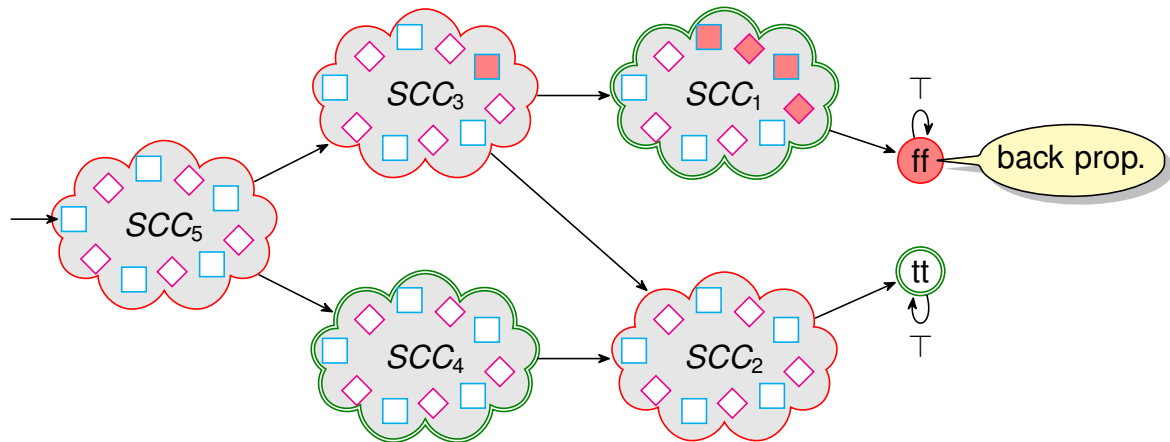
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Solving Weak Games Bottom-Up

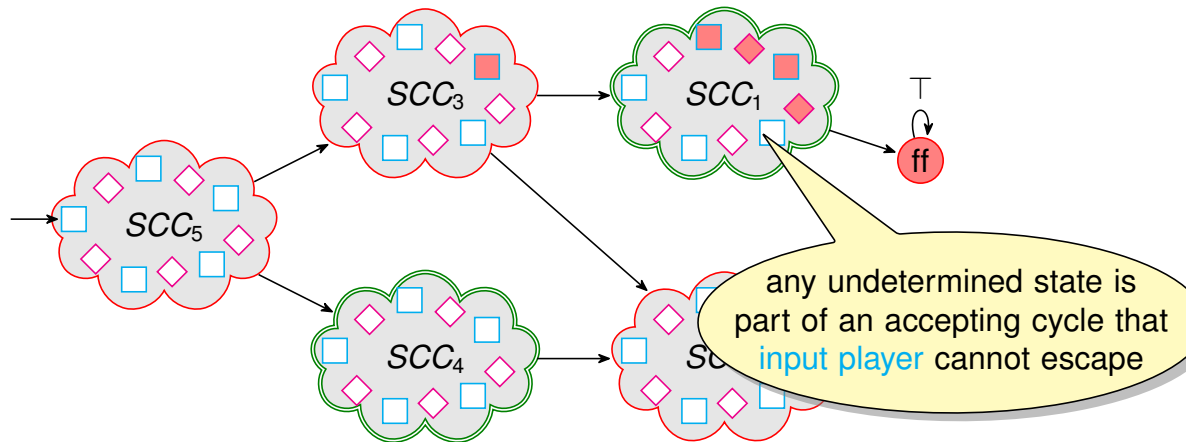
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Solving Weak Games Bottom-Up

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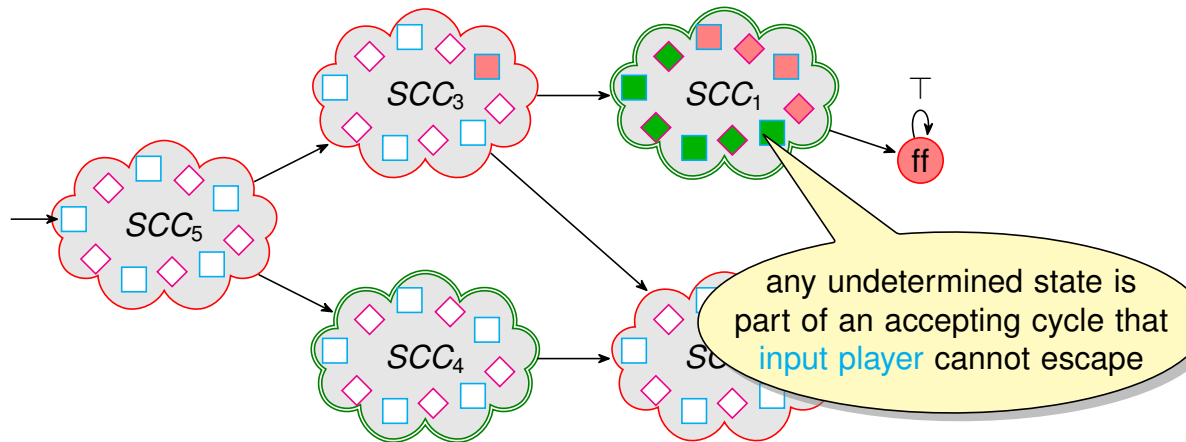
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Solving Weak Games Bottom-Up

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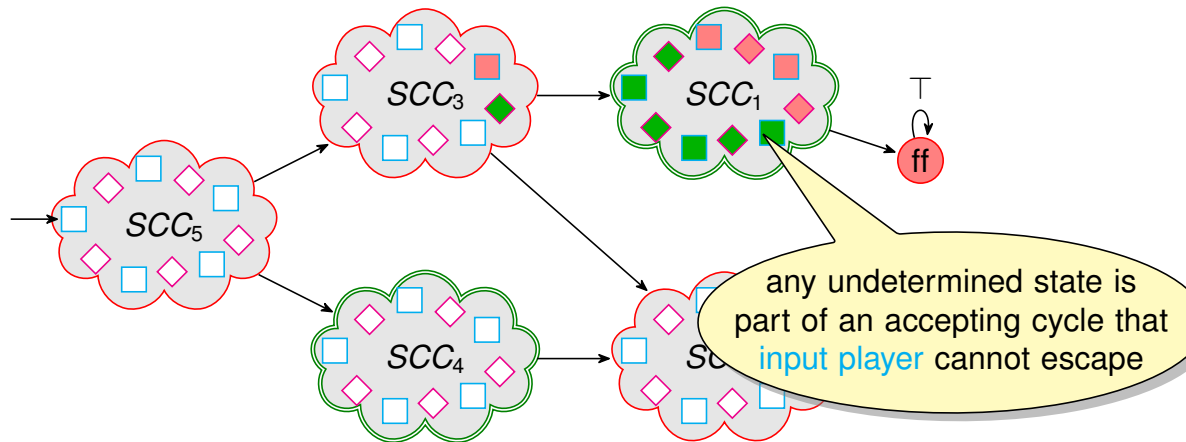
The **output player** wants the play to get stuck in an **accepting SCC**.



Solving Weak Games Bottom-Up

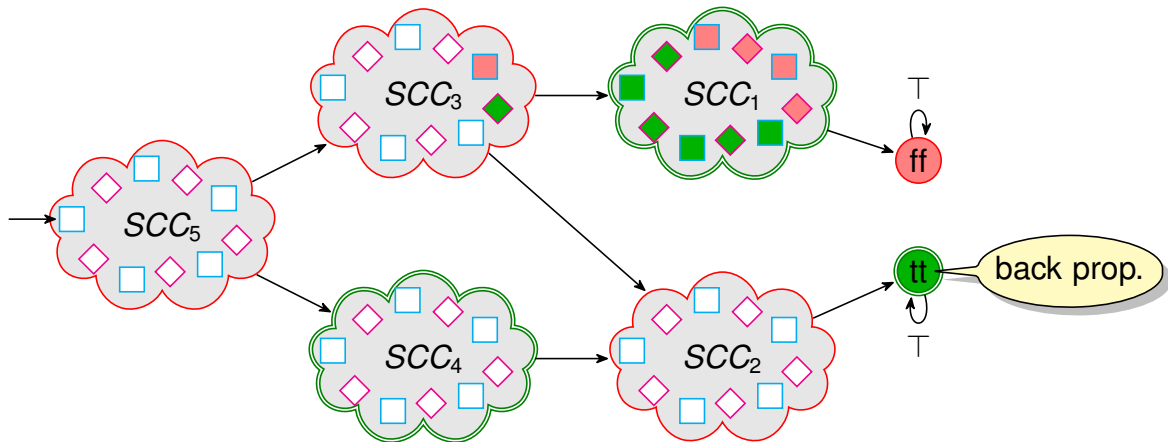
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Solving Weak Games Bottom-Up

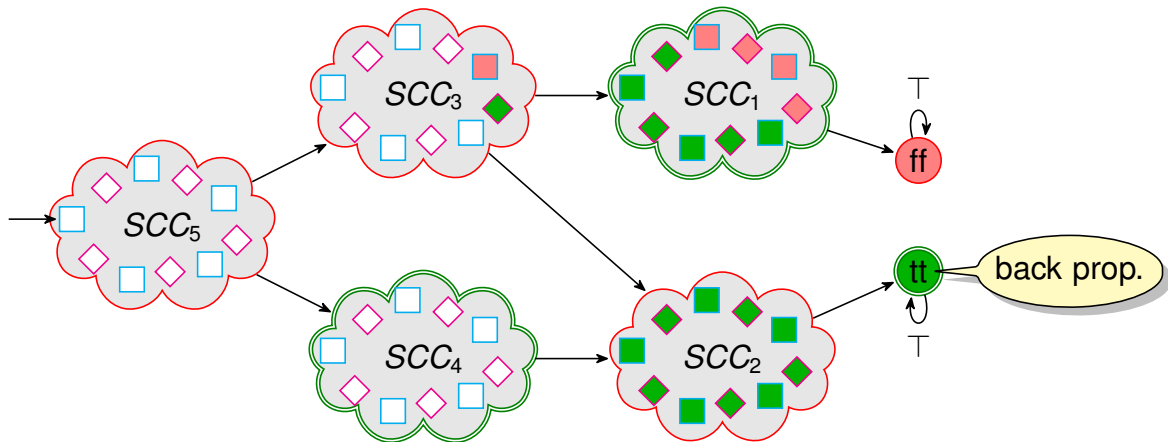
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Solving Weak Games Bottom-Up

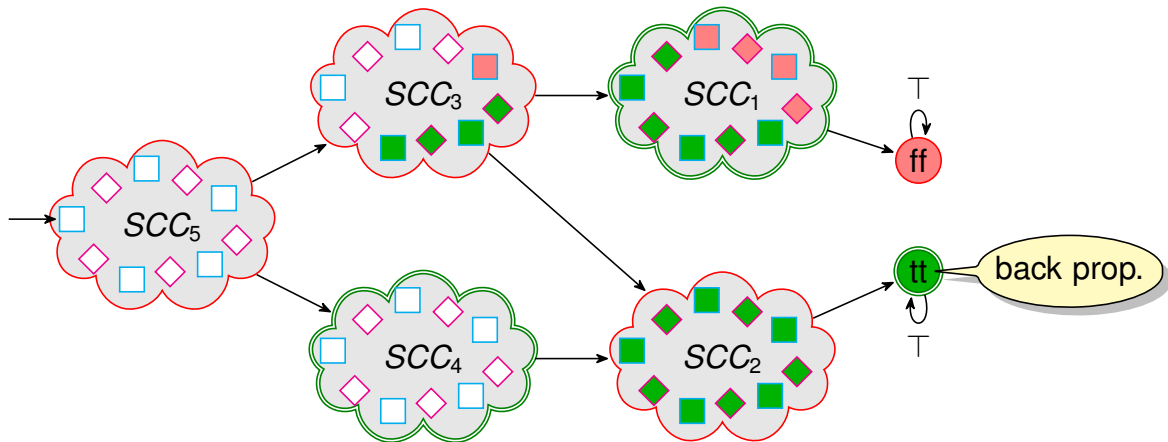
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Solving Weak Games Bottom-Up

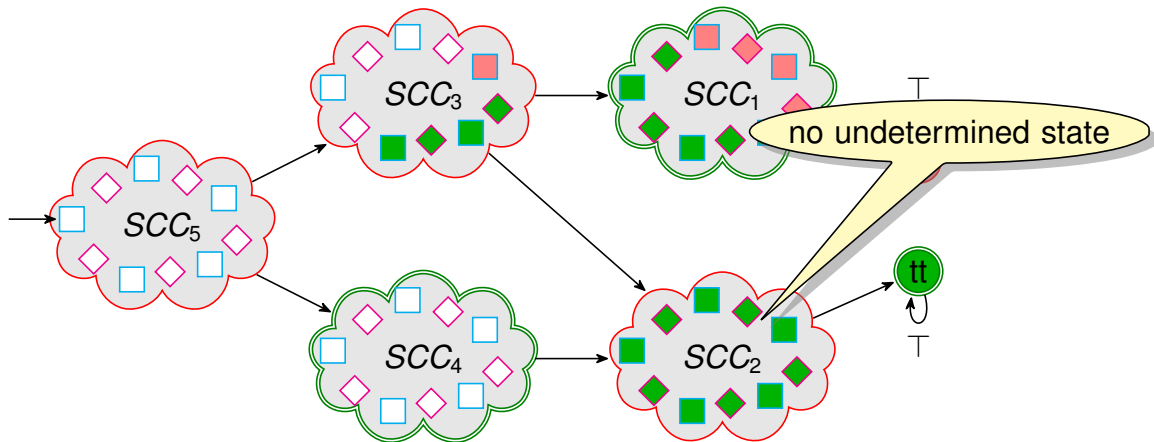
The **input player** wants the play to get stuck in a **rejecting SCC**.
The **output player** wants the play to get stuck in an **accepting SCC**.



Solving Weak Games Bottom-Up

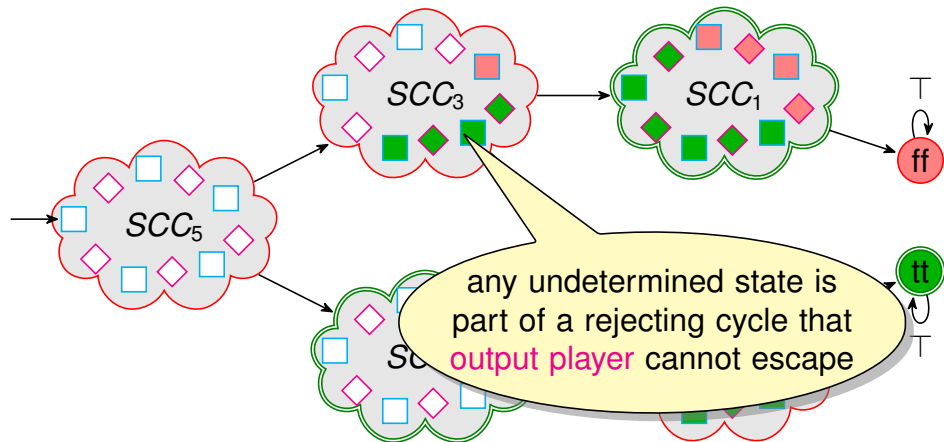
The **input player** wants the play to get stuck in a **rejecting SCC**.

The **output player** wants the play to get stuck in an **accepting SCC**.



Solving Weak Games Bottom-Up

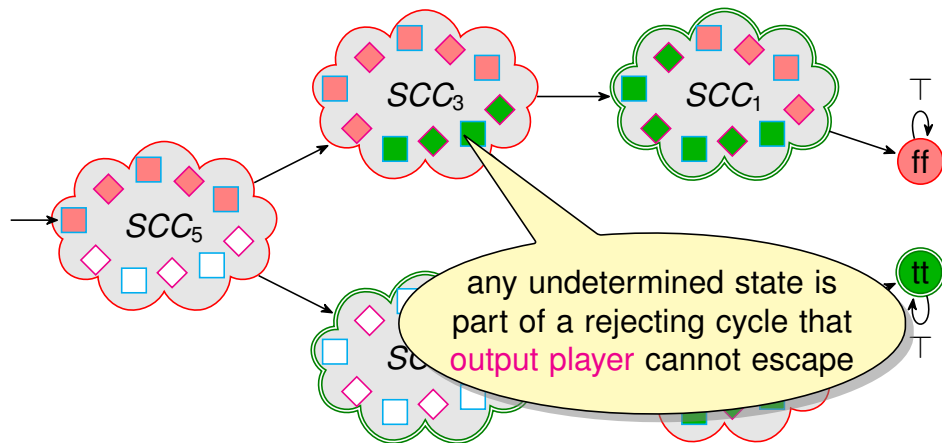
The **input player** wants the play to get stuck in a **rejecting SCC**.
The **output player** wants the play to get stuck in an **accepting SCC**.



Solving Weak Games Bottom-Up

The **input player** wants the play to get stuck in a **rejecting SCC**.

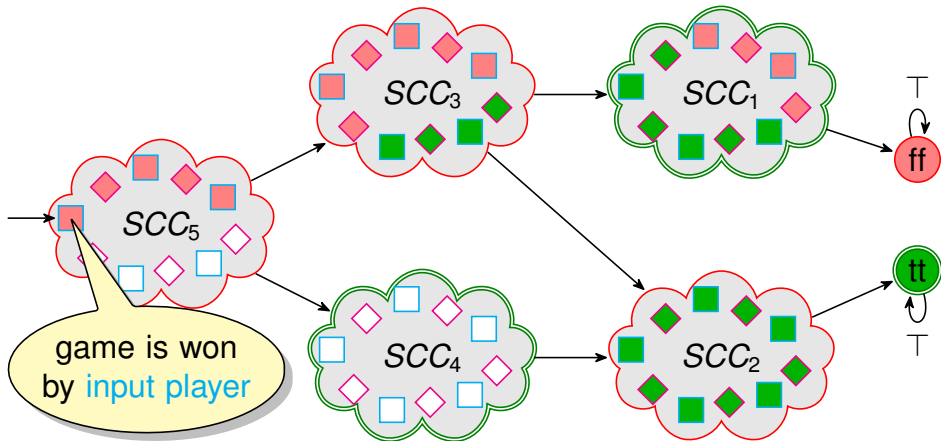
The **output player** wants the play to get stuck in an **accepting SCC**.



Solving Weak Games Bottom-Up

The **input player** wants the play to get stuck in a **rejecting SCC**.

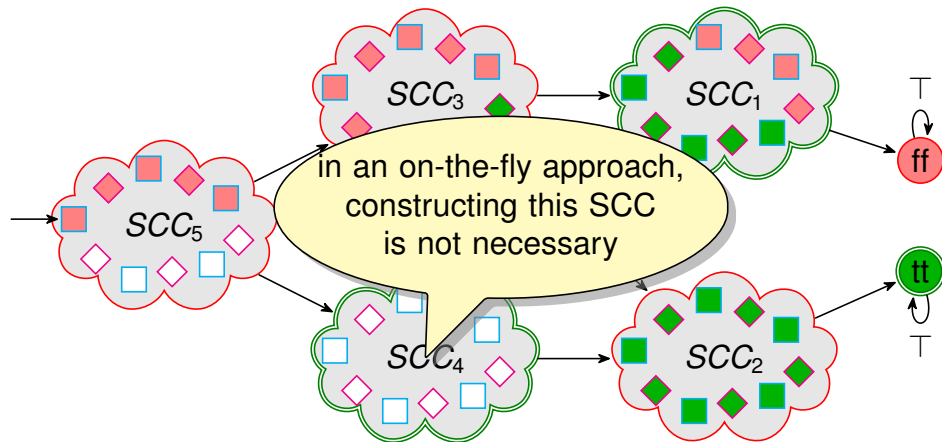
The **output player** wants the play to get stuck in an **accepting SCC**.



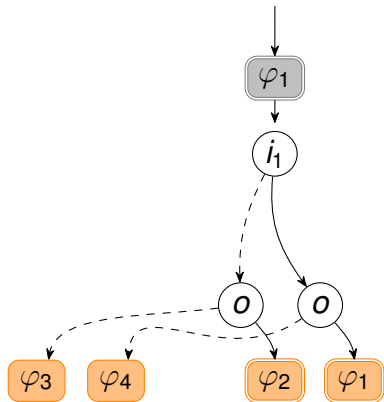
Solving Weak Games Bottom-Up

The **input player** wants the play to get stuck in a **rejecting SCC**.

The **output player** wants the play to get stuck in an **accepting SCC**.



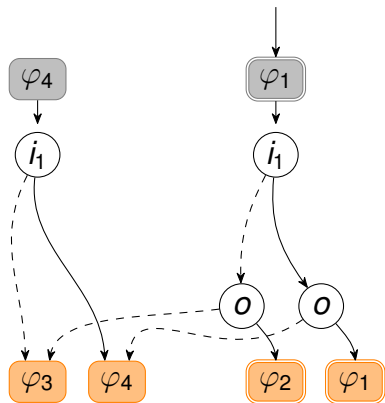
On-the-Fly MTBDD-based Obligation Synthesis



$\{\varphi_1\}$

DFS stack organized
as partial SCCs

On-the-Fly MTBDD-based Obligation Synthesis

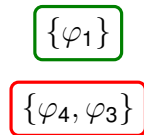
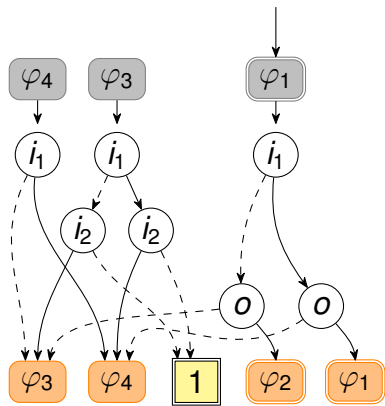


$\{\varphi_1\}$

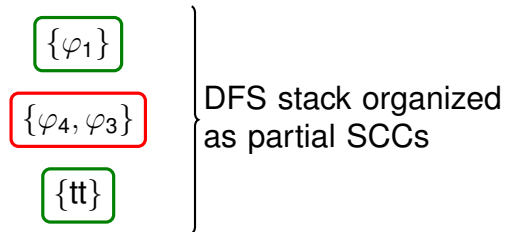
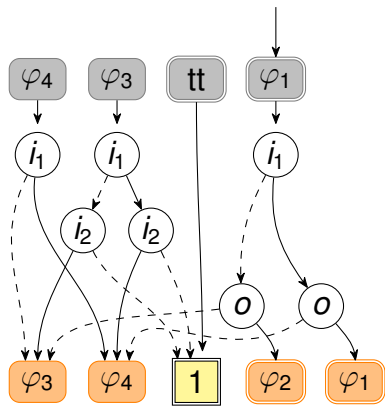
$\{\varphi_4\}$

DFS stack organized
as partial SCCs

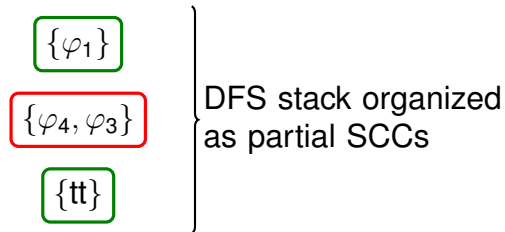
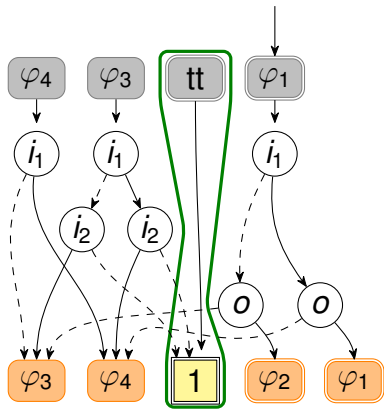
On-the-Fly MTBDD-based Obligation Synthesis



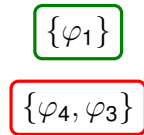
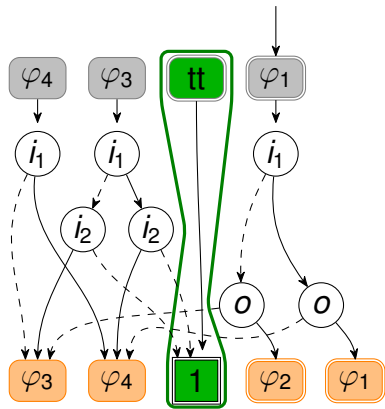
On-the-Fly MTBDD-based Obligation Synthesis



On-the-Fly MTBDD-based Obligation Synthesis

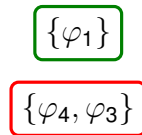
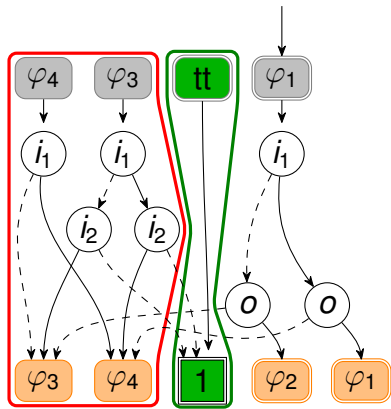


On-the-Fly MTBDD-based Obligation Synthesis



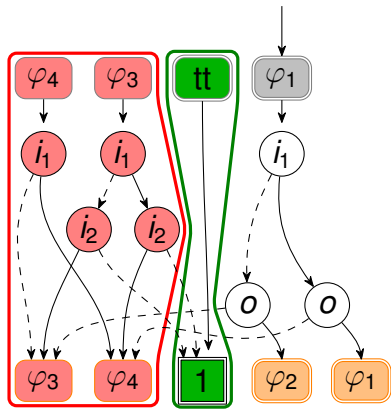
DFS stack organized
as partial SCCs

On-the-Fly MTBDD-based Obligation Synthesis



DFS stack organized
as partial SCCs

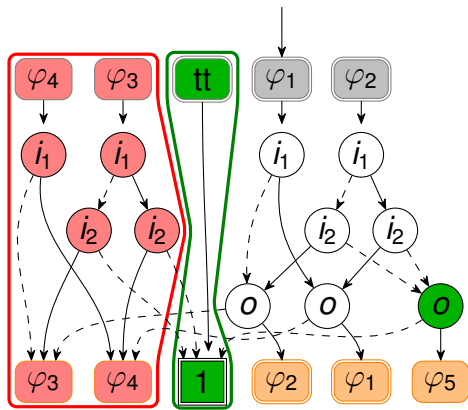
On-the-Fly MTBDD-based Obligation Synthesis



$\{\varphi_1\}$

DFS stack organized
as partial SCCs

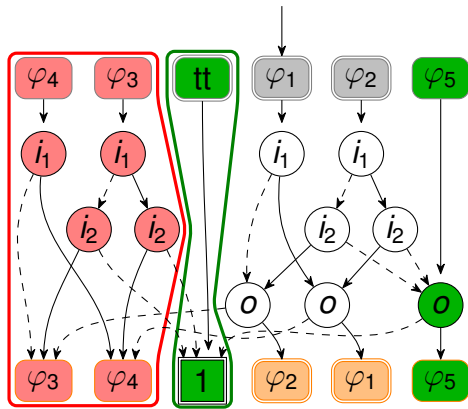
On-the-Fly MTBDD-based Obligation Synthesis



$\{\varphi_1, \varphi_2\}$

DFS stack organized
as partial SCCs

On-the-Fly MTBDD-based Obligation Synthesis

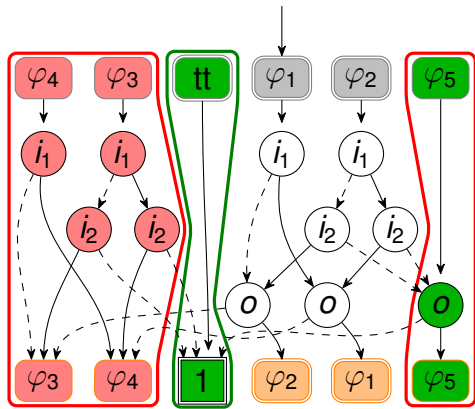


$\{\varphi_1, \varphi_2\}$

$\{\varphi_5\}$

DFS stack organized
as partial SCCs

On-the-Fly MTBDD-based Obligation Synthesis

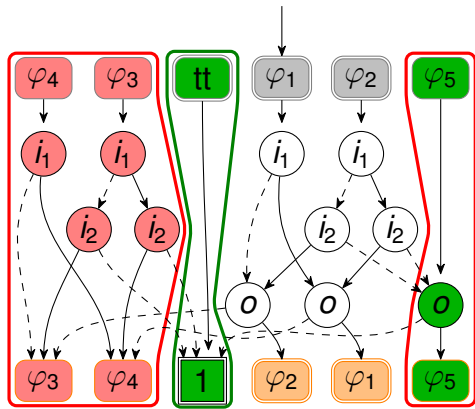


$\{\varphi_1, \varphi_2\}$

$\{\varphi_5\}$

DFS stack organized
as partial SCCs

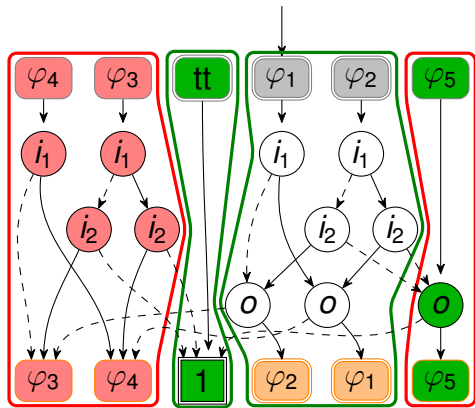
On-the-Fly MTBDD-based Obligation Synthesis



$\{\varphi_1, \varphi_2\}$

DFS stack organized
as partial SCCs

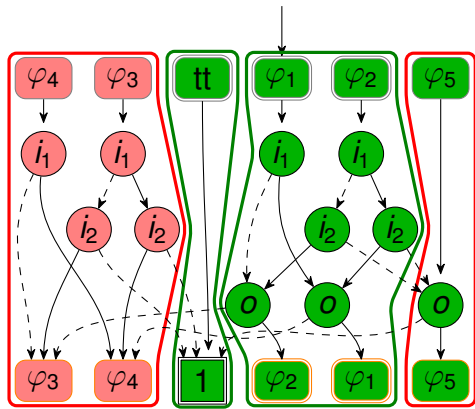
On-the-Fly MTBDD-based Obligation Synthesis



$\{\varphi_1, \varphi_2\}$

DFS stack organized
as partial SCCs

On-the-Fly MTBDD-based Obligation Synthesis



DFS stack organized
as partial SCCs